

Simple models of evolution and extinction

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Abstract

This article gives a brief introduction to the mathematical modeling of large-scale biological evolution and extinction. We give three examples of simple models in this field: the coevolutionary avalanche model of Bak and Sneppen, the environmental stress model of Newman, and the increasing fitness model of Sibani, Schmidt, and Alstrøm. We describe the features of real evolution which these models are intended to explain and compare the results of simulations against data drawn from the fossil record.

1 Introduction

Throughout the 3 billion year history of life on the Earth the processes of evolution and extinction have been inextricably linked. Species survive on average only about 10 million years before they become extinct, so that almost every species that has ever lived is extinct today. This high turnover of species has played a crucial role in long-term evolution because it is the removal of one species which makes way for the evolution of another. The classic example is that of the dinosaurs, whose extinction at the end of the Cretaceous period 65 million years ago cleared the way for the subsequent dominance of the mammals and eventually the evolution of the human race.

Most of our knowledge about prehistoric life comes from the fossil record. Traditionally, fossil studies have focused on the evolution of individual species or groups of species, or on prominent prehistoric events such as mass extinctions or adaptive radiations (the evolution and spread of species to occupy new niches in the ecosystem). However, in the last ten years or so, with the availability of extensive computer databases of fossil species, researchers have also started to look at large-scale patterns in the fossil record, such as the distribution of the sizes of extinction events, and the distribution of the lifetimes of species or groups of species. These studies have led to the suggestion of a variety of new mechanisms which may affect evolution and extinction on long time scales, and of mathematical models incorporating these mechanisms which can mimic some aspects of the development of life. In the following we describe some of the patterns seen in the fossil data and some of the models which have been proposed to explain them.

2 The fossil data

Currently available databases of the fossil record represent about a quarter of a million species, mostly marine animals, which are usually grouped either into genera or into families (the two levels of the Linnean hierarchy immediately above species). The reason for this grouping is that there are not enough fossils of most individual species to make meaningful estimates of when they first appeared and when they became extinct. By grouping them into genera and families we increase the number of fossils per group and thereby the accuracy of origination and extinction dates.

Dating is usually done to the nearest stratigraphic stage. Stages are irregular intervals of time of average duration of about seven million years, which are based on easily identifiable geological features. Almost all the available fossil data come from the Phanerozoic eon, the last 540 million years, during which multicellular life has dominated the planet. There are 77 stages in the Phanerozoic.

One of the most striking proposals that has been put forward in the last few years is that some distributions of fossil quantities may follow power laws in which the probability $p(x)$ of measuring a value x for a particular quantity satisfies

$$p(x) \sim x^{-\alpha}. \quad (1)$$

When such a distribution is plotted on logarithmic scales, one obtains a straight line

$$\log p(x) \sim -\alpha \log x, \quad (2)$$

with slope $-\alpha$.

Figure 1 shows a histogram of the number of families becoming extinct per stratigraphic stage on a log-log scale. The horizontal axis measures the number x of families that became extinct in any given stage of the Phanerozoic, and the vertical axis measures the number of stages in which x took that value. The histogram is clearly skewed heavily to the right—there are many stages in which a few families became extinct, and few in which many became extinct. It has been suggested²⁻⁴ that this histogram follows a power law with a slope of about -2 . In the inset of Figure 1 we show a histogram of the lifetimes of genera, which also appears to follow a power law,^{5,6} with a slope in this case of about $-\frac{3}{2}$. In the following sections we look at some simple models which have been proposed as possible explanations for the generation of power laws such as these.

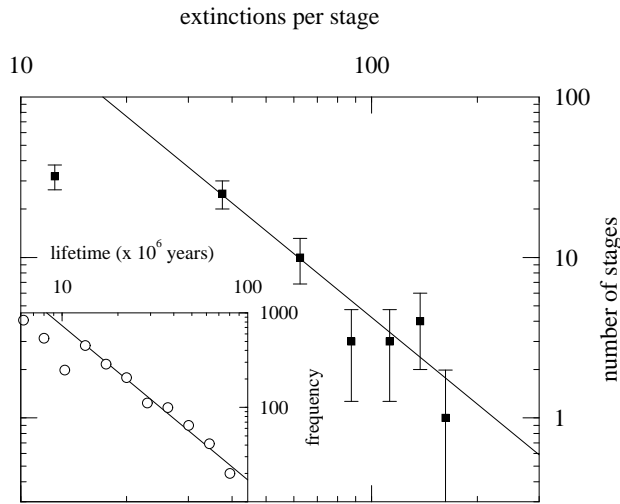


Figure 1: Histogram of the number of families of fossil marine species becoming extinct per stratigraphic stage of the Phanerozoic. Inset: histogram of the lifetimes of genera in the fossil record. The data are taken from the compilation by Sepkoski.¹

3 The model of Bak and Sneppen

The model which has probably generated the most excitement in this field, and which must be credited with stimulating a large part of the recent interest in evolution modeling within the computer simulation community, is the self-organized critical evolution model of Bak and Sneppen.⁷ The basic idea behind this model is that of the *fitness landscape*.

It was the influential British biologist Sewall Wright who first proposed that evolution be viewed as a combinatoric optimization process on a rugged landscape,⁸ similar to the satisfiability problems of computer science⁹ or spin glasses in physics.¹⁰ Organisms or species can be thought of as having a scalar “fitness,” usually denoted by W , which measures their reproductive success. Species with higher reproductive success have more offspring in the next generation and dominate over species with lower reproductive success. For every possible genotype of an organism, that is, for every possible sequence of its DNA, there is an associated value of W which is the fitness of the organism that has that gene sequence. The mapping from genotype to fitness is the fitness landscape. The landscape exists in a very high dimensional space similar to the state space of a physical system such as a spin system.

Evolution serves to move species on the fitness landscape. Because species with higher fitness are favored over those with lower fitness, a mutant strain of organism which finds itself at a higher point on the fitness landscape will dominate over its ancestral strain and over time the population will shift to the fitter genotype. Thus, under the

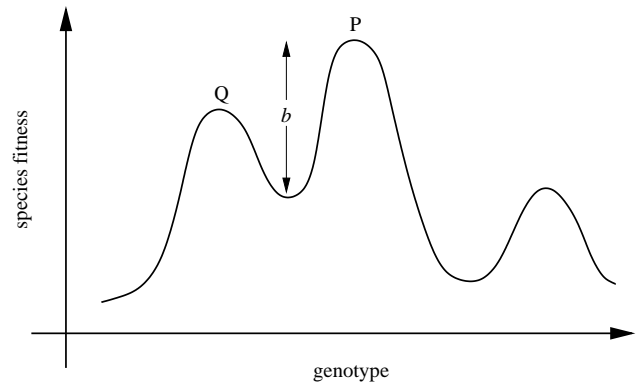


Figure 2: A schematic representation of the “fitness barrier” b which a species at peak P must surmount in order to evolve to a new fitness peak Q . Note that the new peak need not be higher than the old one.

influence of repeated mutation and selection, species tend to move “uphill” on the fitness landscape, only stopping when they reach a local maximum or peak on the landscape. The peaks on a landscape represent all the possible stable species. (Ideas inspired by this view of evolution have been used to formulate new optimization methods in computer science. These methods typically go by the name of *genetic algorithms*¹¹ or *genetic programming*.¹²)

Life would be boring in an ecosystem in which all species simply walked uphill on their own fitness landscape until they reached a local peak. Once everyone found their peak, evolution would stop. This situation is called *Nash equilibrium*. There are a variety of reasons why this situation does not happen in real evolution. First, there may be perturbations from the environment, pressures such as changing climate or changing food supply, which affect the shape of the fitness landscape and force species which were previously stable to evolve into new forms. Even in the absence of such perturbations however, evolution may still occur.¹³ It is possible for a stable population to evolve if one of the members of that population undergoes a large mutation, or a rapid sequence of smaller ones, which moves it so far on the fitness landscape that it finds itself in the basin of attraction of a new fitness peak. Another possibility is that evolution takes place because of interactions between species. Species are not independent; the evolution of one can affect the fitness of another. For example, if you prey on a certain animal which evolves to fly in order to escape you, then you had better evolve to fly too, or learn to eat something else, or you are likely to die out. Thus, the evolution of one species affects the shape of the fitness landscape of the others with which it interacts. This process is called *coevolution*.

Bak and Sneppen⁷ incorporated these ideas into a simple model of evolution as follows. Suppose we have a certain number N of species in an ecosystem, each of which is

localized around a peak on its own fitness landscape. Each species interacts with a number of others, which can be chosen in a variety of ways. The simplest way is to place the species on a lattice and have each interact with its nearest neighbors. Nothing will happen to any of the species as long as they remain at their local peaks. However, every once in a while, a large mutation or sequence of mutations will take a species from its local peak over to the basin of attraction of another peak, and so cause it to evolve. Bak and Sneppen represented the ease with which this “excitation” could take place by a “fitness barrier” b_i for the mutation of the i th species (see Figure 2), analogous to the energy barrier which a physical system has to cross to move from one local energy minimum to another on a rugged energy landscape. The species which has the lowest barrier to mutation is assumed to be the one that evolves first.

Here is where coevolution comes in. When a species evolves by crossing its fitness barrier, it affects the shape of the fitness landscapes of those species with which it interacts. Bak and Sneppen made the simplifying assumption that the fitness landscape of the neighboring species is completely randomized. These neighboring species, which were previously at a comfortable local peak, now find themselves (most probably) not at a peak at all, and so evolve again until they reach a new peak, with a new fitness barrier. This process is represented in the Bak–Sneppen model by choosing a new value at random for the fitness barrier of each neighboring species. But the process stops here: it is assumed that the neighbors of the neighbors do not also evolve. The next species to evolve are the one with the next lowest fitness barrier and its neighbors. Thus the entire model can be summarized as follows:

1. N species are placed on a lattice and each is given a fitness barrier b_i , which initially is chosen at random. Bak and Sneppen used uniform random numbers between zero and one, and this choice seems as good as any.
2. The species with the lowest barrier is found, and its fitness barrier is replaced by a new value, again chosen at random.
3. The nearest neighbors of this species on the lattice are given new random barrier values also.
4. Repeat from step 2.

And that is the entire model. So what does the model do? Well, initially, the dynamics tends to remove all the low-lying barriers from the system and replace them with higher ones, producing a “gap” at the bottom end of the barrier distribution—a range from zero up to some finite value in which none of the barriers fall. However, as time goes by, the gap becomes larger and the probability that a new randomly chosen barrier value falls in this gap increases proportionately. Depending on the coordination number of

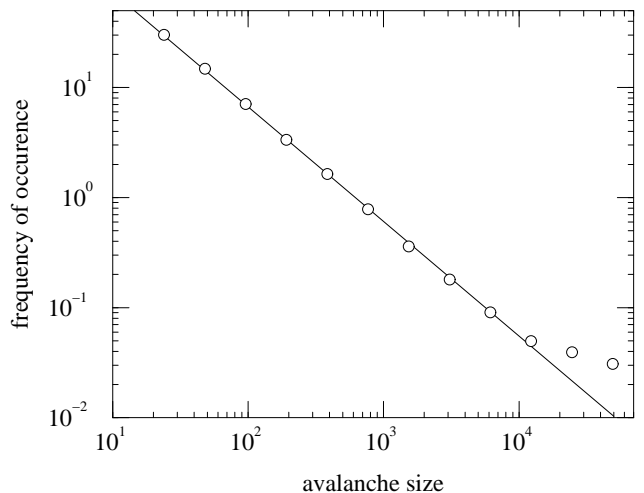


Figure 3: Histogram of the sizes of coevolutionary avalanches in a simulation of the Bak–Sneppen model⁷ with $N = 100$ species on a one-dimensional lattice. The distribution is close to power-law in form and for this simulation has a measured exponent of -1.04 ± 0.01 (the solid line).

the lattice (the number of nearest neighbors of each site), the system will reach a critical point where each time one species is removed from the gap we put another one in, and the system reaches a dynamic equilibrium in which the gap no longer grows.

Bak and Sneppen observed the lengths of the sequences of moves from the moment when a species appears in the gap until the last one is removed. (It is usually fairly clear from the distribution of barrier values where the edge of the gap is—the distribution drops off very sharply there. As Bak and Sneppen showed however, you can obtain good results even if you only get the position of the edge approximately correct.) These sequences they called *coevolutionary avalanches*, a name adopted from the writings of Kauffman.² These avalanches are, in a sense, all the result of one initial evolutionary event in which a species spontaneously mutates to a new genotype which has a barrier value which falls in the gap at the bottom of the distribution. As the gap becomes larger, the lengths of the avalanches increase, until, at the critical point, the average avalanche length diverges, resulting in a scale-free (that is, power-law) distribution of avalanche sizes. Bak and Sneppen speculated that a power-law distribution of coevolutionary avalanches could be the cause of a power-law distribution of extinction sizes in the fossil record: when many species evolve to new forms, all the ancestral forms die out, causing a mass extinction.

In Figure 3 we show a histogram of the sizes of avalanches in a simulation of the Bak–Sneppen model after it has come to equilibrium and indeed we do see that the

distribution has the form of a power law. In this case the simulation was performed on a one-dimensional lattice and the exponent of the power-law is about -1 . This exponent varies with the dimensionality of the lattice, but never gets steeper than $-\frac{3}{2}$, which is still some way from the value of -2 estimated from the fossil data.⁴ This difference is one of the main drawbacks of the Bak–Sneppen model. Another is that the mechanism it proposes whereby ancestral species are wiped out *en masse* by their large-scale evolution into new forms is not thought by paleontologists to be a realistic view of what happens in nature. In fact, most mass extinction events are believed to be caused by stresses on the ecosystem coming from external causes, such as drops in sea level,¹⁴ impacts of extraterrestrial bodies,¹⁵ climate change,¹⁶ or changes in the level of oxygen in the oceans.¹⁷ Both this issue, and the issue of the value of the exponent have been addressed by another simple model of extinction proposed by Newman (that’s me).

4 Newman’s extinction model

Newman¹⁸ has proposed a model of extinction that takes an approach diametrically opposite to that of Bak and Sneppen. Where the Bak–Sneppen model assumes that extinction is caused entirely by (co)evolutionary effects, Newman’s model assumes that it is caused entirely by stresses on the ecosystem from external sources. In fact, there is no interaction between species at all in this model. The reason why large numbers of species become extinct simultaneously is not because they interact with one another, but because they all feel the same stresses at the same time.

The model works like this. We again assume a fixed number N of species, each characterized by a single scalar x_i which is the threshold amount of stress that the species can withstand before it becomes extinct. Stress is represented by a noise variable $\eta(t)$, which fluctuates randomly with time t . The source of the stresses is not specified in the model—only the magnitude of the stress matters. The dynamics of the model is simple: if at any time the stress $\eta(t)$ is numerically greater than the threshold x_i that species i can withstand, then this species becomes extinct at time t . The niches vacated by extinct species are repopulated by new ones which have randomly chosen thresholds x_i . The distribution of the values of stress η is usually chosen to be some decreasing function of η , so that large stresses are less common than small ones.

In fact, this is not quite all there is to Newman’s model. If it were, then the dynamics of the model would stagnate quickly once all the species with low thresholds were removed, leaving only those species with thresholds sufficiently high that they cannot easily be reached by stresses of typical size. To prevent this happening, Newman also included an evolution mechanism in the model, whereby species are occasionally chosen at random and their thresh-

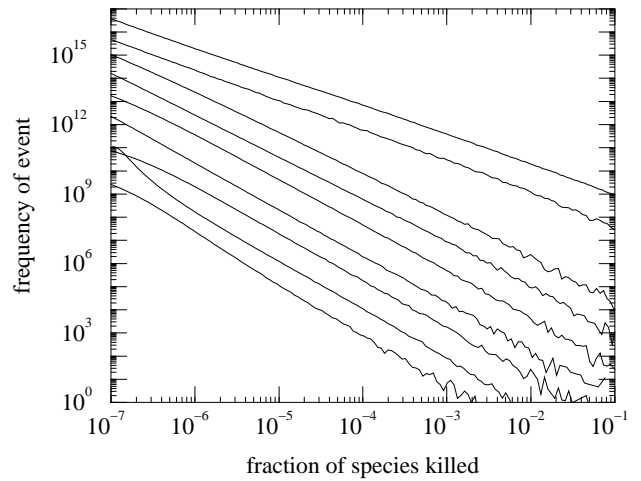


Figure 4: The distribution of the sizes of extinction events in Newman’s model¹⁸ for a variety of different types of applied stress including Gaussian centered around zero, Gaussian centered away from zero, Poissonian, exponential, stretched exponential, and Lorentzian.

olds changed to new randomly chosen values. This mechanism means that there is always an influx of new species with low thresholds to feed the extinction process.

The model can be summarized as follows:

1. Each of the N species is given a threshold value which is initially chosen at random, usually from a uniform distribution between zero and one.
2. A random number η is chosen from some distribution $p_{\text{stress}}(\eta)$ to represent the current stress level. All species i for which $x_i < \eta$ are wiped out and are replaced by new species with randomly chosen thresholds x_i (which may be less than η).
3. A small fraction f of the species are picked at random and “evolved,” meaning that their threshold variables are changed to new randomly chosen values.
4. Repeat from step 2.

The only remaining parameters to be fixed are the value of f and the distribution p_{stress} . In fact, it turns out that the principal predictions of the model do not depend on these choices, within reason. The value of f should be small. Typical values are on the order of 10^{-4} or less. The model equilibrates slower for smaller values, but the results produced are cleaner. The effect of different choices for p_{stress} is illustrated in Figure 4 where we show the distribution of the sizes of extinction events in the model—the number of species that become extinct per time step—for a variety of common noise distributions, including Gaussian noise, Poissonian noise, and power laws. As the figure shows, the distribution of event sizes closely follows a power law,

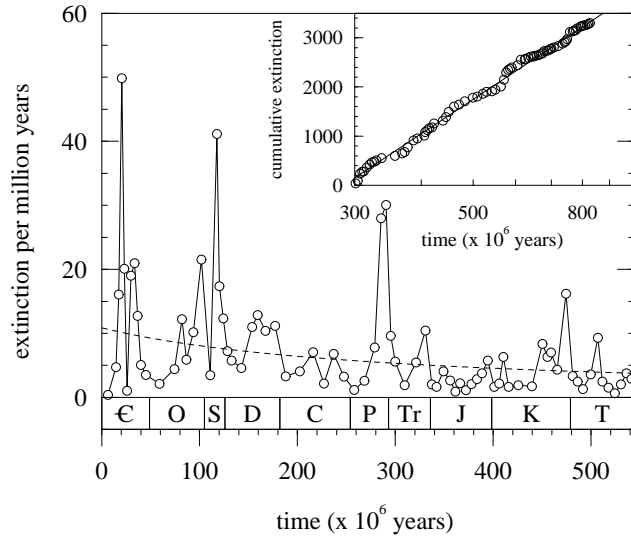


Figure 5: Extinction rate as a function of time during the last 540 million years. The dotted line is Eq. (4). Inset: the cumulative extinction, which appears approximately as a straight line on the linear-log scale used here. The data are taken from the compilation by Sepkoski.¹

with an exponent of about -2 for all of these distributions. Sneppen and Newman¹⁹ have explained this result using an approximate mean-field-like treatment of the model. It is possible to choose a distribution of the applied stresses that will not produce a power-law extinction size distribution (a uniform distribution between zero and one will not, for example), but the cases shown in Figure 4 cover most of the distributions likely to be found in nature.

Newman’s model fits in better with the conventional wisdom within the paleontology community about the causes of extinction events than does the Bak–Sneppen model, and also produces an exponent for the extinction size distribution which is close to that observed in the fossil record. However, it too has its shortcomings. One of these, which we address next, is that, like the Bak–Sneppen model, Newman’s model is a model of an equilibrium world in which the average behavior of the ecosystem does not change over time. This, as we now discuss, is not the case in real life.

5 The model of Sibani, Schmidt, and Alstrom

In Figure 5 we show the rate of extinction of families measured in each stratigraphic stage as a function of time from 540 million years ago to the present. As we can see, the average extinction rate appears to decline towards the present. There are significant fluctuations about this trend—a number of mass extinctions are visible, for instance, as large peaks in graph—but overall there is a clear decline. This

trend is believed to be a real effect—species are living longer and becoming extinct more slowly now than they were a few hundred million years ago (ignoring recent anthropogenic extinctions). In the inset to Figure 5, we show a plot of the *cumulative extinction*, that is, the total number of families (in this case) that disappear from the data set between its start and a given time t as a function of t . The plot has a logarithmic time axis and when plotted in this way the data fall on a very nice straight line. This plot implies that the cumulative extinction takes the form²⁰

$$c(t) = A + B \log(t - t_0), \quad (3)$$

and the extinction rate $r(t)$, which is the derivative of $c(t)$, satisfies

$$r(t) = \frac{B}{t - t_0}. \quad (4)$$

Thus the average extinction rate is clearly not constant in time, as the models of Bak and Sneppen and of Newman implicitly assume. In fact, it declines quite sharply.

What implications does this behavior have for the distributions of quantities such as the sizes of extinction events? The interval of time Δt in which $r(t)$ falls between r and $r + \Delta r$ is given by

$$\Delta t = \frac{dt}{dr} \Delta r, \quad (5)$$

and the number of stages or other intervals of time in which the extinction rate lies in a certain range is proportional to this same expression, that is, proportional to the derivative

$$\frac{dt}{dr} = -\frac{B}{(t - t_0)^2} = -\frac{B}{r^2}. \quad (6)$$

In other words, if the extinction rate satisfies Eq. (4), then the distribution of extinctions in short time intervals such as stages follows precisely the power law with exponent -2 suggested for the fossil record. This explanation of the power law is not perfect, because it assumes that extinction takes precisely the form (4), when in fact this form is only an average. More importantly, it really only passes the buck. It explains one power law (the distribution of the sizes of extinction events) by assuming another (the decline in extinction rate). What is the explanation for this second power law? A simple model explaining this behavior has been proposed by Sibani, Schmidt, and Alstrøm.^{21,22}

The model of Sibani *et al.* is, like the Bak–Sneppen model, based on the idea of evolution on a fitness landscape. Again we consider species to be populations of organisms localized around peaks on the landscape. And as before, these populations are considered to be, by and large, stable. They change only when a mutation or sequence of mutations takes place which is large enough to take them to the basin of attraction of a new peak. In this model there is no coevolution—the species are considered to be non-interacting as in Newman’s model—but there is one

subtlety which is included that is not present in the Bak–Sneppen model. If a single individual in a population has a mutant genotype that puts it in the basin of attraction of a new peak, then the descendents of that individual may well evolve toward that new peak. However, if the fitness at that peak is lower than the fitness at the peak currently occupied by the rest of the population, then the mutant population will not supersede the original one, and no evolution or extinction will take place. Only if the new peak is higher than the original one will the population move and the original species become extinct.

In the model of Sibani *et al.*, this process is represented in a very simple fashion. Each of N species has a fitness W_i . The process of mutation to a new peak is represented by generating a random number r_i for each species i to represent the height of the peak. If $r_i > W_i$, then the species evolves and the ancestral species becomes extinct. Otherwise, nothing changes. And that is the entire model. We can summarize it as follows:

1. For each of our N species we choose an initial real fitness value W_i at random. It turns out that it does not matter from what distribution we choose these numbers. The standard thing is to choose them uniformly between zero and one.
2. At each time step we choose N new random numbers r_i . All species for which $r_i > W_i$ become extinct, and are replaced by descendent species which have $W_i = r_i$.
3. Repeat from step 2.

A process of this type is referred to as *record dynamics*.²³ It is the dynamics one would expect of world records for any quantity if the values of that quantity fluctuate at random (which they usually don't).

In Figure 6, we show the results of a simulation of this model with $N = 10000$ species. The figure has the same layout as Figure 5: the main figure shows a histogram of the actual extinction intensity on linear scales, along with the proposed $1/t$ fit; the inset shows the cumulative extinction. As we can see, the results follow the $1/t$ form closely, and the cumulative extinction makes an excellent straight line on the linear-log scales used, just as in the fossil record. It is not difficult to see why this should be the case. Consider a single species, which after some time t_0 has fitness W_0 . How long will it take before we generate a random number which is higher than this value? On average, it will take the same amount of time that it took to generate this number the first time, which is t_0 . Thus the next evolutionary event will take place after a total time $t_1 = 2t_0$. Repeating the argument, the next one after that will happen at time $t_2 = 2t_1 = 4t_0$, and so on. In general the n th event will happen at around $t = 2^n t_0$. The number of events Δn happening

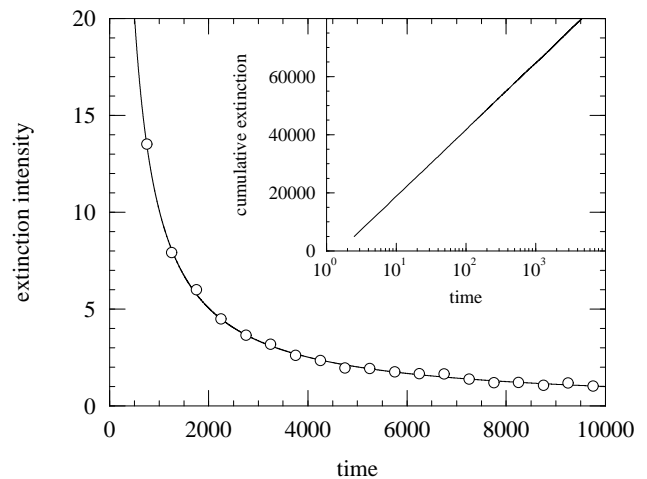


Figure 6: The extinction intensity as a function of time for the model of Sibani *et al.*²¹ with $N = 10000$. The points are simulation results and the solid line is the expected $1/t$ form. Inset: the cumulative extinction, which appears as a straight line on linear-log scales.

in an interval of time Δt will then be

$$\Delta n = \frac{\Delta t}{t \log 2}. \quad (7)$$

In other words, the extinction rate falls as $1/t$.

This model, like the others we have discussed, has its problems. Chief among them is the fact that, like the Bak–Sneppen model, it assumes that all extinction is caused by descendent species superseding their ancestors. For the case of the large mass extinction events, this is almost certainly not the true cause of extinction; these events are believed to have been caused by environmental stress. However, smaller “background” extinction events do not, by and large, have known causes, so the model of Sibani *et al.* is perhaps plausible as a model of background extinction.

6 Conclusions

We have outlined three simple models of evolution and extinction which attempt to explain some of the features seen in the fossil record. The model of Bak and Sneppen is a model of extinction caused by large-scale coevolution—the evolution of one species in response to that of another. This model is a self-organized critical model that displays “avalanches” of coevolutionary activity whose size is distributed according to a power law. Newman has proposed a contrasting model in which extinction is caused by external stresses on the ecosystem. In this model, species do not interact at all, but the model still shows a power-law distribution of the sizes of extinction events. In the model of Sibani, Schmidt, and Alstrøm, species evolve when they

generate a mutant strain that is fitter than its parent. This evolution produces an ever-increasing species fitness, with jumps, which are associated with extinction events, occurring less and less frequently over time. This process also gives rise to a power-law distribution of extinction events.

So which of these models is right? Certainly none of them tell the whole story. Each one offers a possible explanation of some feature of the fossil record, but each one leaves out many things as well. It is quite conceivable that all of the mechanisms in these models are occurring simultaneously in nature and combining to give the signatures we see in the fossil data. Or maybe none of them are. There is a lot of activity in this field at the moment, and new mechanisms and models are being proposed all the time. Models based on ecological interactions, on the structure of food webs, on competition between species for resources, and on many other principles are currently under investigation. Ref. 24 gives an extensive review of recent work. In the long run, it is hoped that further simulations, along with detailed analyses of the fossil data, will help us to discover the processes that were at work during the evolution of life on the Earth.

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